An Equivalent Modeling Method for Multi-port Area Load Based on the Extended Generalized ZIP Load Model

Yuan Ji, Xi Zhang, Xiangdong Wang, Xiaoxu Huang, Bin Huang, J. H. Zheng, *Member, IEEE*, Zhigang Li, *Member, IEEE*

***Abstract*—To model the load area and simplify the electrical network, a multi-port area load equivalent modelling method (ALEEM) based on the extended generalized ZIP load model (EGZIP) is proposed. Different from the traditional ZIP load, the EGZIP load model incorporates the voltage magnitudes and voltage phase angles of all boundary buses, which can equivalently model the area load with multiple boundary buses more accurately. The load flow calculation considering the EGZIP is derived and analyzed. Also, based on the hierarchical and partitioning characteristics of the power grid, a multi-port equivalent strategy is proposed to reduce the number of parameters to be identified in the equivalent model. For parameter identification, the currents measured at varying operation conditions on the boundary bus are used to construct the least square estimation (LSE) problem. The interior point method is used to identify the model parameters. The simulation test conducted on the 87 bus system proves the equivalent model derived from the proposed ALEEM based on EGZIP has higher accuracy and the multi-port equivalence strategy can reduce both the number of parameters to be identified and the time consuming on equivalent process.**

***Index Terms*—area load, equivalent modeling, parameter identification, multi-port, ZIP load**

I. Introduction

In order to improve the quality of electric power and the reliability of power transmission and distribution, the modern power grid has developed into a large-scale interconnected system, which brings new challenges to the system analysis. In the operation and scheduling of transmission network, considering the detailed model of the full network including the load area, it will readily encounter the problem of dimension disaster, which not only significantly increases the computing burden, but also is not conducive to acquiring the real-time status of the power grid [1]. In order to tackle this issue, the network equivalent method can be used to model the load area, so as to reduce the scale of the system and reduce the complexity of the problems such as scheduling, planning and static security analysis [2].

The goal of the network equivalent method is to reduce the scale of the power grid by simplifying the external network as much as possible and at the same time keeping the consistency of the analysis results of the equivalent network and the original network. Generally, it is believed that the network equivalent method can be divided into two kinds [3]. One is the topology based methods, and among them, the most widely used are Ward equivalent [4] and its improved version [5-6] and REI (Radial Equivalent Independent) equivalent [7] and its improved version. The other kind of network equivalent method is measurement-based methods. Normally, in this kind of method, the black box model is used to simulate the external network, which means that the topology of the external network is unknown. The input signals are introduced in black box model and corresponding output signals of the black box model can be obtained by the PMU (phasor measurement units). In order to make the output of the equivalent model as close to the output of the real system as possible, the parameters of the black box model need to be identified. Therefore, the measurement-based method is essentially a parameter identification method [8]. Considering that the area load consist of numerous and various loads, it is very difficult to obtain the detailed topology of the load area and the power flow information. Usually, The transmission network is equipped with numerous measurement device in the transformer substation [9]. Hence, the measurement based methods are promising in the equivalent modelling of area load.

The first step in the area load equivalent modelling is to determine the structure of the equivalent model. A reasonable equivalent model structure is the key to the accuracy of the equivalent model [10]. The accuracy of the equivalent model depends on its structure. Reference [11] presents an N+1 bus

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structure based extended Thevenin equivalent method for real-time voltage stability monitoring for load areas, which adopts the constant impedance equivalent load model. Two kinds of external network equivalent models are proposed in [12], which are the simplified Ward equivalent model and the extended voltage source model. With regard to the modelling of area load, [13] adopts the equivalent model composed of fictitious buses, fictitious branches and ZIP (constant impedance, constant current, constant power) loads at the boundary bus. This model takes the static voltage characteristics of the load into account. However, the introduction of the fictitious buses will bring difficulties to the model parameter identification, thus compromising the accuracy of the equivalent model. Reference [14] utilizes the equivalent model in which boundary buses are interconnected through fictitious branches. The ZIP load model is used in both [13] and [14]. The ZIP load model is characterized by that the load of the bus is determined by the nodal voltage magnitude. However, under the scenario where the area load with multiple boundary buses, the load should be determined by the complex voltage of the set of boundary buses jointly. Hence, the ZIP load model has limitations in tackling the equivalent modelling of area load.

The next step in the area load equivalent modelling is to determine the parameters of the equivalent model. Some researchers regard the measurement equation as the objective function and treat the unknown parameter of the equivalent model as the optimization variables. Then, the least square estimation problem is constructed, and the optimization algorithm is used to identify the unknown parameter of the equivalent model [3], [8], [13], [14].

In this paper, an extended generalized ZIP load model (EGZIP) is proposed and used in area load equivalent modelling method (ALEMM). In ALEMM, the equivalent load at each boundary bus is determined by the voltage magnitude and the differences of the voltage phase angle between all boundary buses. To diminish the number of unknown parameters, a multi-port equivalence strategy is also proposed, which forms the equivalent models of different area loads separately. For parameter identification, the currents measured at different operation conditions and the different time on the boundary bus are used to construct the least square estimation (LSE) problem and the interior point method is used to identify the model parameters.

II. EGZIP LOAD MODEL

A. Mathematical formula of EGZIP model

The area loads are the important part of the power system, and the transmission network supplies the electric power to them through a distribution substation. The area load is usually composed of feed branches, distribution substations, switches, compensators, voltage regulators and loads. The area loads have many characteristics, such as multiple voltage levels, complex network structure and so on. Establishing accurate and valid equivalent models for area load is vital to the analysis and the control of the power grid [14].

In general, as shown in Figure 1, the electrical network with a load area can be divided into three parts: network to be preserved, boundary buses, load area. ALEEM aims to replace the area load with the simpler circuit. Considering the total load consumed of load area is determined by the complex voltage vectors on all boundary buses, EGZIP load model is proposed in this paper to substitute the area load.

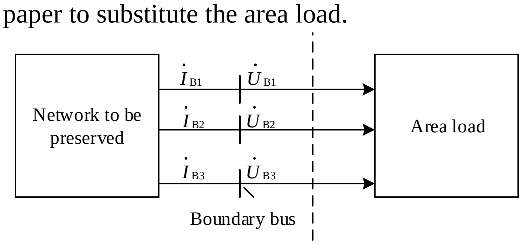

Fig. 1. The electrical network with area load

In the EGZIP load model, the equivalent load at each boundary bus is determined by the voltage magnitude and the differences of voltage phase angle between all boundary buses. Assuming that there are more than two boundary buses, the equivalent active load for boundary bus i is shown as follows:

$$P_i^{\mathrm{eq}} = \frac{1}{2}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_\Delta\end{bmatrix}^T \begin{bmatrix}\boldsymbol{A}_{\mathrm{VV}}^{\mathrm{p}i} & \boldsymbol{A}_{\mathrm{V\theta}}^{\mathrm{p}i}\\ \boldsymbol{A}_{\mathrm{V\theta}}^{\mathrm{p}i\,T} & \boldsymbol{A}_{\theta\theta}^{\mathrm{p}i}\end{bmatrix}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_\Delta\end{bmatrix} + \begin{bmatrix}\boldsymbol{b}_{\mathrm{V}}^{\mathrm{p}i}\\ \boldsymbol{b}_{\theta}^{\mathrm{p}i}\end{bmatrix}^T \begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_\Delta\end{bmatrix} + c^{\mathrm{p}i}, \quad (1)$$

where i is the index of boundary bus.

$\boldsymbol{V}$ is the column vector of voltage magnitude of all boundary buses:

$$\boldsymbol{V} = \begin{bmatrix}V_1 & V_2 & \cdots & V_K\end{bmatrix}^{\mathrm{T}}, \quad (2)$$

where K is the number of boundary buses.

$\boldsymbol{\theta}_\Delta$ is the differences of the voltage phase angles between each pair of boundary buses:

$$\boldsymbol{\theta}_\Delta = \begin{bmatrix}\theta_{21} & \theta_{32} & \cdots & \theta_{K(K-1)}\end{bmatrix}^{\mathrm{T}}, \quad (3)$$

where $\theta_{K(K-1)}$ is the differences of the voltage phase angle between buses K and $K-1$.

$\boldsymbol{A}_{\mathrm{VV}}^{pi}$ is the symmetric coefficient matrix representing relationship between the voltage amplitude of the boundary buses:

$$\boldsymbol{A}_{\mathrm{VV}}^{pi} = \begin{bmatrix} 2a_{(V_1,V_1)}^{pi} & a_{(V_1,V_2)}^{pi} & \cdots & a_{(V_1,V_K)}^{pi}\\ & 2a_{(V_2,V_2)}^{pi} & \cdots & a_{(V_2,V_K)}^{pi}\\ & & \ddots & \vdots\\ & & & 2a_{(V_K,V_K)}^{pi}\end{bmatrix}, \quad (4)$$

where $a_{(V_1,V_K)}^{pi}$ is the quadratic coefficient with regard to V_1 and V_K and it represents an part of nonlinear characteristic of area load, which is related to voltage magnitude of different boundary buses pairs.

$\boldsymbol{A}_{\mathrm{V}\theta_\Delta}^{pi}$ is the quadratic coefficient matrix representing relationship between the voltage magnitude and the differences of the voltage phase angles:

$$\boldsymbol{A}_{\mathrm{V}\theta_\Delta}^{pi}=\begin{bmatrix} a_{(V_1,\theta_{21})}^{pi} & \cdots & a_{\left(V_1,\theta_{K(K-1)}\right)}^{pi} \\ a_{(V_2,\theta_{21})}^{pi} & \cdots & a_{\left(V_1,\theta_{K(K-1)}\right)}^{pi} \\ \vdots & \vdots & \vdots \\ a_{(V_K,\theta_{21})}^{pi} & \cdots & a_{\left(V_K,\theta_{K(K-1)}\right)}^{pi} \end{bmatrix}, \tag{5}$$

where $a_{\left(V_K,\theta_{K(K-1)}\right)}^{pi}$ is the quadratic coefficient with regard to V_K and $\theta_{K(K-1)}$, and it represents an part of nonlinear characteristic of area load, which is related to the voltage magnitude and the differences of the voltage phase angle.

$\boldsymbol{A}_{\theta_\Delta\theta_\Delta}^{pi}$ is the symmetric coefficient matrix representing relationship between the differences of the voltage phase angle the boundary buses :

$$\boldsymbol{A}_{\theta_\Delta\theta_\Delta}^{pi}=\begin{bmatrix} 2a_{(\theta_{21},\theta_{21})}^{pi} & \cdots & a_{\left(\theta_{21},\theta_{K(K-1)}\right)}^{pi} \\ & \ddots & \vdots \\ & & 2a_{\left(\theta_{K(K-1)},\theta_{K(K-1)}\right)}^{pi} \end{bmatrix}, \tag{6}$$

where $a_{\left(\theta_{21},\theta_{K(K-1)}\right)}^{pi}$ is the quadratic coefficient with regard to θ_{21} and $\theta_{K(K-1)}$, and it represents an part of nonlinear characteristic of area load, which is related to the difference of voltage angle of the boundary buses.

$\boldsymbol{b}^{pi}$ is the linear coefficient vector:

$$\boldsymbol{b}^{pi}=\begin{bmatrix} b_{V_1}^{pi} & b_{V_2}^{pi} & \cdots & b_{V_K}^{pi} & b_{\theta_{21}}^{pi} & \cdots & b_{\theta_{K(K-1)}}^{pi} \end{bmatrix}^{\mathrm{T}}, \tag{7}$$

where $b_{V_K}^{pi}$ is the linear coefficient with regard to V_K and $b_{\theta_{K(K-1)}}^{pi}$ is the linear coefficient with regard to $\theta_{K(K-1)}$.

The expression for the equivalent reactive load is similar to that of the equivalent active load:

$$Q_i^{\mathrm{eq}}=\frac{1}{2}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_\Delta\end{bmatrix}^{\mathrm{T}}\boldsymbol{A}^{qi}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_\Delta\end{bmatrix}+\boldsymbol{b}^{qi\,\mathrm{T}}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_\Delta\end{bmatrix}+c^{qi}, \tag{8}$$

It is noted that with singular boundary bus, the differences of the voltage phase angle do not exist any more and EGZIP load can be degenerated into traditional ZIP load, indicating EGZIP load is the extended version of traditional ZIP load.

B. Comparison between EGZIP and ZIP

The formulation of the traditional ZIP load is shown as follows:

$$P_i^{\mathrm{eq}}=a_1^{\mathrm{p}i}V_i^2+a_2^{\mathrm{p}i}V_i+a_3^{pi}, \tag{9}$$

$$Q_i^{\mathrm{eq}}=a_1^{\mathrm{q}i}V_i^2+a_2^{\mathrm{q}i}V_i+a_3^{\mathrm{q}i}, \tag{10}$$

where a_1^{pi}, a_2^{pi} and a_3^{pi} are the constant coefficients for the constant impedance, the constant current, and the constant power components of the active power equivalent load, respectively. The definition of coefficient for the reactive power equivalent load is similar.

Different from the traditional ZIP load, the EGZIP load is determined by the voltage magnitude and variances of the phase angle of all boundary buses jointly, instead of the voltage magnitude of specific boundary bus merely. From the angle of the mathematical model, EGZIP preserves the quadratic, linear and constant terms with respect to both voltage magnitude and voltage angle. Hence, EGZIP can depict the nonlinear characteristics of the area loads more effectively.

It is noted that with a single boundary bus, the differences of the voltage phase angles between boundary buses do not exist any more and EGZIP load can be degenerated into traditional ZIP load, indicating EGZIP load is the extended version of traditional ZIP load.

C. Load flow calculation embedded with EGZIP

In the EGZIP load model, both the active load and the reactive load are the function of the voltage magnitude of the boundary buses and the differences of the voltage phase angles between boundary buses. In load flow calculation, some changes are necessary considering the compatible issue.

Taking the Newton Raphson method as an example, first of all, when the power imbalance is calculated in the iteration process, the active and reactive power load of the bus are updated using the value from (1) and (8), and then the updated loads are used for the next iteration.

Furthermore, the matrix elements in the Jacobian matrix of the power flow equation should add the partial derivatives of active power with regard to voltage magnitude $\partial P_i^{\mathrm{eq}}/\partial V_i$ and the partial derivatives of active power with regard to voltage magnitude $\partial Q_i^{\mathrm{eq}}/\partial V_i$:

$$\partial P_i^{\mathrm{eq}}/\partial V_i=\boldsymbol{A}_{i\bullet}^{pi}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_\Delta\end{bmatrix}+b_{V_i}^{pi}, \tag{11}$$

where $\boldsymbol{A}_{i\bullet}^{pi}$ is the i th row of $\boldsymbol{A}^{pi}$, $b_{V_i}^{pi}$ is the element of $\boldsymbol{b}^{pi}$ corresponding to V_i.

The derivation of $\partial Q_i^{\mathrm{eq}}/\partial V_i$ is similar to that of $\partial P_i^{\mathrm{eq}}/\partial V_i$.

If there is only one boundary bus, because there is no difference of the voltage phase angles between the boundary buses, the matrix elements related to the partial derivative of power with regard to the voltage angle in the Jacobian matrix of the power flow equation need not change. If the boundary nodes are two or more, the matrix elements in the Jacobian matrix of the power flow equation should add the partial derivative of power with regard to the voltage phase angle $\partial P_i^{\text{eq}}/\partial\theta_i$ and $\partial Q_i^{\text{eq}}/\partial\theta_i$.

If the number of boundary buses are two:

$$\partial P_i^{\text{eq}}/\partial\theta_j=\begin{cases}-\boldsymbol{A}_{(K+1)\cdot}^{pi}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_{\Delta}\end{bmatrix}-b_{\theta_{K(K-1)}}^{pi},\ j=1\\ \boldsymbol{A}_{(K+1)\cdot}^{pi}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_{\Delta}\end{bmatrix}+b_{\theta_{K(K-1)}}^{pi},\ j=2\end{cases},\qquad(12)$$

where j is the index of boundary bus and K is the number of boundary buses.

If the number of boundary buses are three or more:

$$\partial P_i^{\text{eq}}/\partial\theta_j=\begin{cases}-\boldsymbol{A}_{(K+1)\cdot}^{pi}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_{\Delta}\end{bmatrix}-b_{\theta_{21}}^{pi},\ j=1\\ \boldsymbol{A}_{(j-1+K)\cdot}^{pi}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_{\Delta}\end{bmatrix}+b_{\theta_{j(j-1)}}^{pi}-\boldsymbol{A}_{(j+K)\cdot}^{pi}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_{\Delta}\end{bmatrix}+b_{\theta_{(j+1)j}}^{pi},\ j\neq 1,K\\ \boldsymbol{A}_{(2K-1)\cdot}^{pi}\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{\theta}_{\Delta}\end{bmatrix}+b_{\theta_{K(K-1)}}^{pi},\ j=K\end{cases},\qquad(13)$$

The derivation of $\partial Q_i^{\text{eq}}/\partial\theta_i$ is similar to that of $\partial P_i^{\text{eq}}/\partial\theta_i$.

III. MULTI-PORT ALEEM BASED ON THE EGZIP MODEL

A. ALEEM based on the EGZIP model

Fig. 2 depicts the equivalent electrical network with EGZIP load, which can substitute the original electrical network in Fig. 1.

As shown in Figure 3, the characteristic of the equivalent model used in [3] is the introduction of a fictitious bus, and the voltage of the fictitious bus is calculated from different boundary buses. As shown in Figure 4, the boundary buses of the equivalent model used in [14] are interconnected through fictitious branches, and the introduction of fictitious bus is avoided. In the following passage, the equivalent model in [14] is called M1, and the equivalent model in [3] is called M2.

B. Parameters identification

The unknown parameters to be identified in the equivalent electrical network are $\boldsymbol{A}^i$, $\boldsymbol{b}^i$ and c^i. The least square estimation method is employed to identify them:

$$\min F(\mathbf{A}^i,\mathbf{b}^i,c^i)=\sum_{j=1}^{J}\sum_{i=1}^{K}\left|I_{\text{ic}}^{j}-I_{\text{is}}^{j}\right|^2,\qquad(14)$$

where I_{is}^{j} is the injected current of boundary bus sampled from measurement equipment, I_{ic}^{j} is the corresponding calculated value derived from EGZIP model and J is the number of operation states.

The derivation of I_{ic}^{j} is as follow:

$$I_{\text{ic}}^{j}=\left(P_i^{\text{eq}}+\text{j}Q_i^{\text{eq}}\Big/\dot{U}_{\text{is}}^{j}\right)^{*},\qquad(15)$$

where P_i^{eq} and Q_i^{eq} are derived from (1) and (8), $\dot{U}_{\text{is}}^{j}$ is the measurement data of volatge of bus i under operation state j and $*$ denotes the conjugate operator.

In this the least square estimation method, (14) is the objective function, (1), (8) and (15) are the constraints. To solve this parameters identification problem, interior point method can be used.

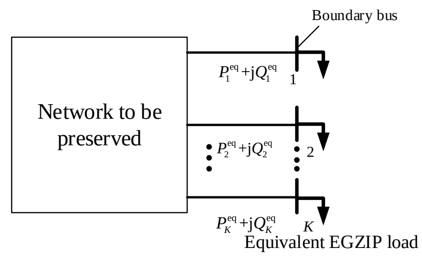

Fig. 2. The equivalent electrical network with EGZIP load

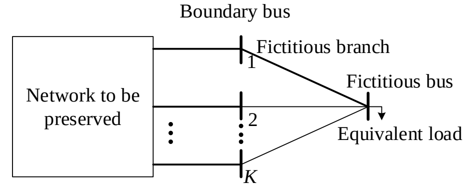

Fig. 3. The equivalent model in [3]

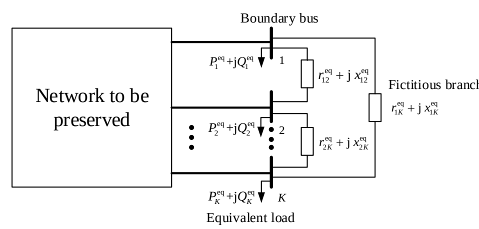

Fig. 4. The equivalent model in [14]

C. Multi-port equivalence strategy

To diminish the unknown parameters, the multi-port equivalence strategy is adopted, the core of which are to equivalence different load areas separately. Accordingly, the dimensions of the unknown parameters is reduced greatly and the complexity of the least square estimation optimization model is assuaged. Take the network in Fig. 5 as example. The network has two area loads, corresponding to port 1 and port 2, respectively. For area load X, the equivalent active load can be expressed using EGZIP:

$$P_i^{\text{eq}}=\frac{1}{2}\begin{bmatrix}\boldsymbol{V}_X\\ \boldsymbol{\theta}_{\Delta X}\\ \boldsymbol{V}_Y\\ \boldsymbol{\theta}_{\Delta Y}\end{bmatrix}^{\text{T}}\begin{bmatrix}A_{XX}^{\text{pi}} & A_{XY}^{\text{pi}}\\ A_{YX}^{\text{pi}} & A_{YY}^{\text{pi}}\end{bmatrix}\begin{bmatrix}\boldsymbol{V}_X\\ \boldsymbol{\theta}_{\Delta X}\\ \boldsymbol{V}_Y\\ \boldsymbol{\theta}_{\Delta Y}\end{bmatrix}+\begin{bmatrix}b_X^{\text{pi}}\\ b_Y^{\text{pi}}\end{bmatrix}^{\text{T}}\begin{bmatrix}\boldsymbol{V}_X\\ \boldsymbol{\theta}_{\Delta X}\\ \boldsymbol{V}_Y\\ \boldsymbol{\theta}_{\Delta Y}\end{bmatrix}+c^{pi},\qquad(16)$$

where the subscript X and Y are corresponding to area load X and Y, respectively.

In combined equivalence strategy, as (16) shows, the number of parameters to be identified are $4(m+n)^3+2(m+n)^2$. However, in multi-port equivalence strategy, matrixes or vectors A_{XY}^{pi}, A_{YX}^{pi}, A_{YY}^{pi}, and b_{Y}^{pi} are null because there is no direct topological connection between these two area loads. Hence, the the number of parameters to be identified are reduced to $4m^3+2m^2+4n^3+2n^2$.

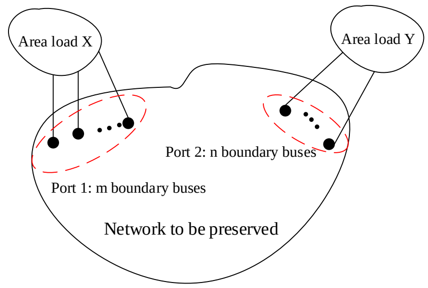

Fig. 5. The network with two area loads

IV. SIMULATION STUDIES

To demonstrate the effectiveness of the proposed model and the multi-port equivalence strategy, the simulation studies are carried out on the 87 bus test system.

The 87 bus test system consists of the IEEE30 bus system and the IEEE57 bus system. The two systems are connected through Bus 7 in the 30 bus system and Bus 4 in the 57 bus system. The test system has 87 buses, 13 generators and 122 branches. Detailed model data can be found in the MATPOWER toolbox [15].

In this system, two area loads can be divided. The detailed grouping of area loads can be seen in Table 1.

To verify the effectiveness of the multi-port equivalent strategy in reducing the unknown parameters of the equivalent model. Both the combined equivalent strategy and the multi-port equivalent strategy are adopted to model the area loads in 87 bus system, respectively. The effects of the strategy are shown in Table 2. The parameters to be identified using the multi-port equivalent strategy are 252, which is 73% less than those of the combined equivalent strategy. Concerning the CPU time, ALEMM using the multi-port equivalent strategy consumed 2.06250s, which is 19% less than that of combined equivalent strategy. This shows that the multi-port equivalent strategy can greatly reduce the parameters of the equivalent model, then reduce the complexity of the least square estimation optimization problem, and finally reduce the time required for the equivalent modelling.

Using the multi-port ALEMM based on EGZIP load model, an equivalent system with 68 buses and 95 branches can be obtained.

In order to verify the accuracy of the equivalent model based on the EGZIP load model and demonstrate the validity and robustness of ALEMM. Regarding the load flow information of 87 bus system as the benchmark, the proposed equivalent model with EGZIP load is compared with the model 1 and the model 2 under the three typical operation states.

TABLE I
THE GROUPING OF AREA LOADS

Area load	Boundary buses	The buses in area load
1	10, 15, 27	18-26,29,30
2	52, 59, 62	53-58,60,61

TABLE II
THE EFFECTS OF EQUIVALENCE STRATEGY

Srategy	The number of parameters	Time (s)
combined	936	2.54687
multi-port	**252**	**2.06250**

The following are the details of three typical operation states:

OS 1: the branch 6-28 belonging to the 30 bus system and the branch 71-72 belonging to the 57 bus system are out of service;

OS 2: The total active power output of all generators in the network to be preserved increased by 10%, and all the load increased by 10% as well.

OS 3: The generator connecting bus 5 in the 30 bus system and the generator connecting bus 36 in the 57 bus system are out of service;

In the tests, the maximum and average relative error of the nodal voltage magnitude of the equivalent network, and the maximum and average relative error of the complex branch flow is used to measure the accuracy of the equivalent model, which are defined as follow:

$$e_{\mathrm{V}}^{\max}=\max_{\forall i\in N}(\left|V_i-V_i^{\mathrm{eq}}\right|/\left|V_i\right|)\times 100\% \ , \tag{17}$$

$$e_{\mathrm{V}}^{\mathrm{avg}}=\sum_{i=1}^{N}(\left|V_i-V_i^{\mathrm{eq}}\right|/\left|V_i\right|)\Big/N\times 100\% , \tag{18}$$

$$e_{\mathrm{S}}^{\max}=\max_{\forall n\in L}(\left|\dot{S}_n-\dot{S}_n^{\mathrm{eq}}\right|\Big/\left|\dot{S}_n\right|)\times 100\% \ , \tag{19}$$

$$e_{\mathrm{S}}^{\mathrm{avg}}=\sum_{n=1}^{L}(\left|\dot{S}_n-\dot{S}_n^{\mathrm{eq}}\right|\Big/\left|\dot{S}_n\right|)\Big/L\times 100\% , \tag{20}$$

where i is the index of the bus, N is the number of buses in equivalent network and L is the number of branches in equivalent network.

Fig. 6 - Fig. 8 show the errors of the proposed equivalent model, model 1 and model 2. It can be seen from the figures that the accuracy of the proposed equivalent model is higher than those of model 1 and model 2 under aforementioned three typical operation states, not matter from the angle of voltage magnitude or from the angle of branch flow. For example, under OS2, the average relative error of the voltage magnitude of the proposed equivalent model is 0.000018, which is 40.0% less than that of model 1 and 74.6% less than that of model 2. The maximum relative error of the voltage magnitude of the proposed equivalent model is 0.000386, which is 57.3% less

than that of model 1 and 84.6% less than that of model 2. The average relative error of the branch flow of the proposed equivalent model is 0.009435, which is 75.5% less than that of model 1 and 86.8% less than that of model 2. The maximum relative error of the branch flow of the proposed equivalent model is 0. 346804, which is 18.1% less than that of model 1 and 83.6% less than that of model 2.

The improvement of the accuracy results from that the proposed EGZIP load model in this study not only considers the static voltage characteristics of the load but also considers the phase angle and the correlation of the voltage vector between the boundary buses, which is superior in modelling the area load with multiple boundary buses. In addition, compared with model 1 and model 2, the equivalent model proposed in this paper has more parameters to be estimated and preserves the nonlinearity of the area load, thus it can describe the area load more accurately. However, among these equivalent models, the equivalent model proposed in this paper has the most concise topology.

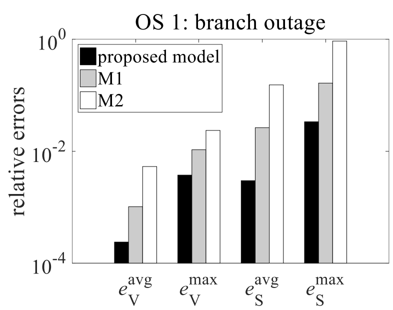

Fig. 6. The relative errors of different equivalent model under OS 1

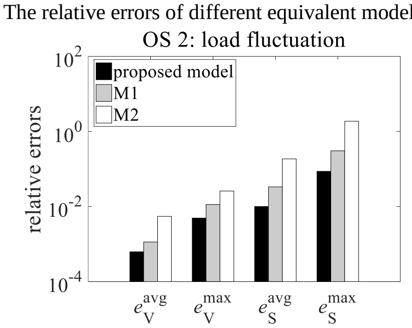

Fig. 7. The relative errors of different equivalent model under OS 2

V. CONCLUSION

This paper proposes ALEMM based on EGZIP load model to construct the equivalent model of power system with area loads. Simulation tests are conducted on the 87 bus system. The test results demonstrate that multi-port equivalent strategy can reduce the parameters to be identified, thereby reducing the complexity of LSE problem and decreasing the consumed time on equivalent modeling. In addition, from the simulation tests under three operation states, it can be seen that the proposed equivalent model derived from ALEEM based on EGZIP has higher accuracy than other equivalent model in terms of the voltage magnitude and branch flow because EGZIP load model considers the mutual relationship of voltage between the boundary buses and it is the generalized version of ZIP load model.

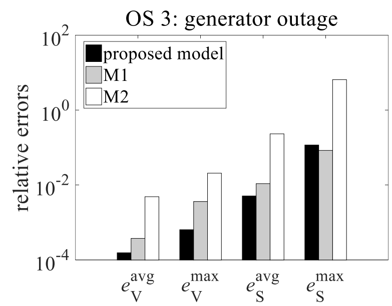

Fig. 8. The relative errors of different equivalent model under OS 3

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