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Letter

A Modified Ward Equivalent Based on Sensitivity Matrices for Static Security Analysis

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This letter proposes a modified Ward equivalent method (M-Ward) for the static security analysis of electric network. The M-Ward develops the external to boundary sensitivity matrices to track the change of the operation condition in the external network. Hence, the M-Ward can guarantee the accuracy of the equivalent network along with the condition change of the external network. Simulation studies of static security analysis conducted on the IEEE 39-bus system demonstrate that the results of M-Ward are more accurate than those of the classic Ward and the extended Ward method. © 2018 Institute of Electrical Engineers of Japan. Published by John Wiley & Sons, Inc.

Keywords: Ward equivalent; external network static equivalent; sensitivity analysis; static security analysis

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1. Introduction

Electric network is a large-scale interconnected system, the static security analysis of which has encountered difficulties such as intractable computation burden and communication latency [1,2]. To overcome these difficulties, external network static equivalent methods (ENSE), including Ward equivalence and its extended versions, are adopted [3]. However, the aforementioned methods are highly state-dependent, which means that a non-negligible error may occur once the operation condition deviates from the basic operation point.

Consequently, this letter develops a modified Ward equivalent method (M-Ward) for the static security analysis of electric network. M-Ward builds the external to boundary sensitivity matrices using the fast decoupled load flow equations. With sensitivity matrices and measured boundary buses voltage, the power injection and voltage on external buses can be updated adaptively, and then, Ward equivalent can be conducted according to the updated data from the external network.

2. Mathematical Implement

Generally, the interconnected electric network is divided into three parts, including internal network, boundary buses and external network, as illustrated in Fig. 1.

Without loss of generality, the power flow equation of interconnected electrical network can be expressed as:

$$\begin{bmatrix} Y_{\mathrm{EE}} & Y_{\mathrm{EB}} & \\ Y_{\mathrm{BE}} & Y_{\mathrm{BB}} & Y_{\mathrm{BI}} \\ & Y_{\mathrm{IB}} & Y_{\mathrm{II}} \end{bmatrix} \begin{bmatrix} \dot{V}_{\mathrm{E}} \\ \dot{V}_{\mathrm{B}} \\ \dot{V}_{\mathrm{I}} \end{bmatrix} = \begin{bmatrix} \dot{I}_{\mathrm{E}} \\ \dot{I}_{\mathrm{B}} \\ \dot{I}_{\mathrm{I}} \end{bmatrix} \tag{1}$$

where Y is the element of bus admittance matrix; $\dot{V}$ is the voltage phasor; $\dot{I}$ is the current injection; and subscript E, B and I denote the element of matrix and vector corresponding to external buses, boundary buses and internal buses, respectively.

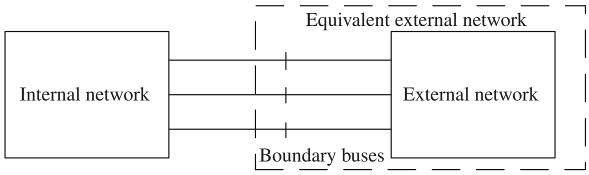

Fig. 1. Interconnected electric network

Performing Gaussian elimination on (1) to eliminate the external buses and transforming bus current injection into bus power injection, the power flow equation of the equivalent network can be expressed as:

$$\begin{bmatrix} Y_{\mathrm{BB}} - Y_{\mathrm{BE}} Y_{\mathrm{EE}}^{-1} Y_{\mathrm{EB}} & Y_{\mathrm{BI}} \\ Y_{\mathrm{IB}} & Y_{\mathrm{II}} \end{bmatrix} \begin{bmatrix} \dot{V}_{\mathrm{B}} \\ \dot{V}_{\mathrm{I}} \end{bmatrix} = \begin{bmatrix} \left(\frac{\dot{S}_{\mathrm{B}}}{\dot{V}_{\mathrm{B}}}\right)^{*} - Y_{\mathrm{BE}} Y_{\mathrm{EE}}^{-1} \left(\frac{\dot{S}_{\mathrm{E}}}{\dot{V}_{\mathrm{E}}}\right)^{*} \\ \left(\frac{\dot{S}_{\mathrm{I}}}{\dot{V}_{\mathrm{I}}}\right)^{*} \end{bmatrix} \tag{2}$$

where $\dot{S}$ is the bus power injection, and * is the symbol of conjugate.

In terms of (2), the classic Ward equivalent (C-Ward) and the extended Ward equivalent (E-Ward) assume the external buses power injection S_{E} and external buses voltage phasor V_{E} are constant, and they are derived from power flow information of basic operation point. However, this assumption will result in enormous errors when the operation point changes.

Aiming to reduce errors, sensitivity matrices can be used to quantify the responses of the external network toward the change of operating condition first, as shown in the Fig. 1, by excluding the internal network from the original network. The equivalent external network (EEN) can be built, where boundary buses are set to slack buses. Thus, variables in the EEN are the phase angle and magnitude of boundary buses voltage θ_{B}, V_{B}; the phase angle of external PV and PQ buses voltage θ_{E}; the magnitude of external PQ buses voltage $V_{\mathrm{E,PQ}}$; and the reactive power injection of external PV buses $Q_{\mathrm{E,PV}}$.

For the EEN, the active power part of XB-type fast decoupled load flow equations can be expressed as:

$$B^{'}\Delta\theta = \begin{bmatrix} B^{'}_{\mathrm{EE}} & B^{'}_{\mathrm{EB}} \\ B^{'}_{\mathrm{BE}} & Y_{\mathrm{BB}} \end{bmatrix} \begin{bmatrix} \Delta\theta_{\mathrm{E}} \\ \Delta\theta_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \frac{\Delta P_{\mathrm{E}}}{V_E} \\ \frac{\Delta P_{\mathrm{B}}}{V_B} \end{bmatrix} \tag{3}$$

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where B' is the coefficient matrix of active power iteration equation, and Δ is the symbol of variation of variables.

Considering the equation related to the $\Delta P_{\mathrm{E}}/V_{\mathrm{E}}$ and ΔP_{E} is zero as external buses are all PV and PQ buses:

$$B'_{\mathrm{EE}}\Delta\theta_{\mathrm{E}} + B'_{\mathrm{EB}}\Delta\theta_{\mathrm{B}} = \frac{\Delta P_{\mathrm{E}}}{V_E} = 0 \tag{4}$$

$$\Delta\theta_{\mathrm{E}} = S_\theta\Delta\theta_{\mathrm{B}}, \qquad S_\theta = -B'^{-1}_{\mathrm{EE}}B'_{\mathrm{EB}} \tag{5}$$

where S_θ is the sensitivity matrix with respect to phase angle.

Similarly, the reactive power part of XB-type fast decoupled load flow equations for the EEN can be expressed as:

$$\begin{bmatrix} B''_{\mathrm{B,B}} & B''_{\mathrm{B,(E,PQ)}} & B''_{\mathrm{B,(E,PV)}} \\ B''_{\mathrm{(E,PQ),B}} & B''_{\mathrm{(E,PQ),(E,PQ)}} & B''_{\mathrm{(E,PQ),(E,PV)}} \\ B''_{\mathrm{(E,PV),B}} & B''_{\mathrm{(E,PV),(E,PQ)}} & B''_{\mathrm{(E,PV),(E,PV)}} \end{bmatrix} \begin{bmatrix} \Delta V_{\mathrm{B}} \\ \Delta V_{\mathrm{E,PQ}} \\ \Delta V_{\mathrm{E,PV}} \end{bmatrix} = \begin{bmatrix} \Delta Q_{\mathrm{B}} \\ \Delta Q_{\mathrm{E,PQ}} \\ \Delta Q_{\mathrm{E,PV}} \end{bmatrix} \tag{6}$$

where B'' is the coefficient matrix of reactive power iteration equation, and the subscript (E,PV) and (E,PQ) denote PV buses and PQ buses in external network, respectively.

Considering $\Delta V_{\mathrm{E,PV}}$ and $\Delta Q_{\mathrm{E,PQ}}$ are zero in external network, ΔV_{B}'s relationship between the $\Delta V_{\mathrm{E,PQ}}$ and $\Delta Q_{\mathrm{E,PV}}$ can be expressed as:

$$\Delta V_{\mathrm{E,PQ}} = S_{\mathrm{V}}\Delta V_{\mathrm{B}}, \qquad S_V = -B''^{-1}_{\mathrm{(E,PQ),(E,PQ)}}B''_{\mathrm{(E,PQ),B}} \tag{7}$$

$$\Delta Q_{\mathrm{E,PV}} = S_{\mathrm{Q}}\Delta V_{\mathrm{B}}, \qquad S_Q = B''_{\mathrm{(E,PV),B}} + B''_{\mathrm{(E,PV),(E,PQ)}}S_V \tag{8}$$

where S_{V} is the sensitivity matrix related to magnitude, and S_{Q} is the sensitivity matrix related to reactive power.

When the operation condition changes greatly, $\dot{S}_{\mathrm{E}}$ and $\dot{V}_{\mathrm{E}}$ can be updated using sensitivity matrix S_{V}, S_{Q}, S_θ and online boundary bus voltage $\dot{V}_{\mathrm{B}}$. Then, (2) can be used to build the new equivalent network with the updated $\dot{S}_{\mathrm{E}}$ and $\dot{V}_{\mathrm{E}}$.

3. Simulation Studies

Static security analysis is conducted on the IEEE 39-bus system to demonstrate the efficiency of proposed M-Ward method. The grouping of the IEEE 39-bus system is as follows:

External buses: Buses 1–2, 25–30 and 37–39;

Boundary buses: Buses 3 and 17;

Internal buses: Buses 4–16, 18–24 and 31–36;

Three cases are considered as follows:

Case 1: The loads and active power output of generators in internal network are increased by 5%.

Case 2: The loads and active power output of generators in internal network are increased by 10%.

Case 3: Branch 4–5 is out of service.

As indices for the accuracy of equivalent network, the relative error e_1 and security error e_2 are defined as follows:

$$e_1 = \left|\frac{x - x_{\mathrm{eq}}}{x}\right| \times 100\%, \qquad e_2 = \left|\frac{x - x_{\mathrm{eq}}}{S_{\mathrm{base}}}\right| \times 100\% \tag{9}$$

where x and x_{eq} are the solutions obtained from the original network and from equivalent network, respectively, and S_{base} is the MVA rating of branch.

Figures 2 and 3 depict the errors of M-Ward, E-Ward and C-Ward for Case 1 and Case 2, respectively. It can be seen from Figs 2 and 3 that the errors of M-Ward are much lower than those of E-Ward and C-Ward under the conditions of load fluctuation. Figure 4 is for Case 3, and it can be further seen that, compared with the results of E-Ward and C-Ward, results of the M-Ward method is more accurate, especially in voltage magnitude and branch reactive power flow, because M-Ward can track the change of the voltage and reactive power injection in the external network using sensitivity matrices.

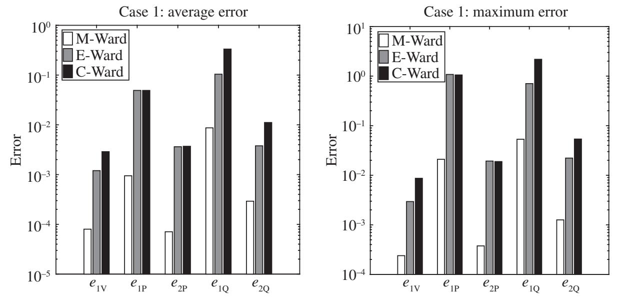

Fig. 2. Error comparisons in Case 1

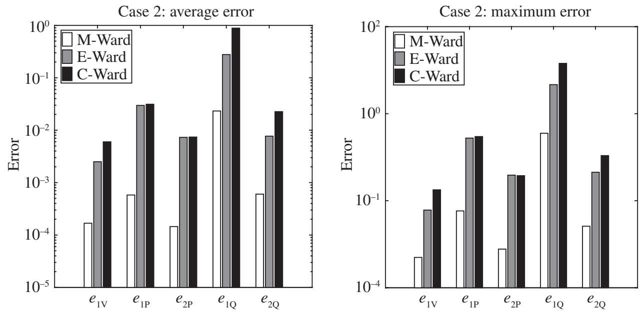

Fig. 3. Error comparisons in Case 2

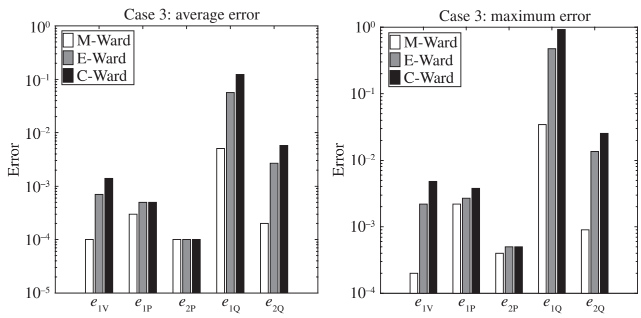

Fig. 4. Error comparisons in Case 3

4. Conclusion

This letter has proposed a modified Ward equivalent for the static analysis of electric network. Sensitivity matrices of the external network are developed to quantify the responses from external network toward the change of operation condition. Simulation results have demonstrated that the performance of the proposed M-Ward for the static security analysis is superior compared with the C-Ward and E-Ward.

Acknowledgment

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